1) Find the total differential.

a) $z = 2x^2y^3$

b) $z = x\cos y - y\cos x$

c) $w = 2z^3 y\sin x$

a) $\boxed{dz = 4xy^3dx + 6x^2y^2dy}$

b) $\boxed{dz = (\cos y + y\sin x)dx - (x\sin y + \cos x)dy}$

c) $\boxed{dw = 2z^3y\cos x\,dx + 2z^3y\sin x\,dy + 6z^2y\sin x\,dz}$

2) If $z = 5x^2 + y^2$ and (x, y) changes from $(1,2)$ to $(1.05, 2.1)$, find the values of Δz and dz.

$$\boxed{\Delta z = 0.9225,\ \ dz = 0.9}$$

3) The radius r and height h of a right circular cylinder are measured with possible errors of 4% and 2%, respectively. Approximate the maximum possible percent error in measuring the volume.

$$\boxed{10\%}$$

4) A triangle is measured and two adjacent sides are found to be 3 inches and 4 inches long, with an included angle of $\frac{\pi}{4}$. The possible errors in measurement are 0.0625 inches for the sides and 0.02 radian for the angle. Approximate the maximum possible error in the computation of the area.

$$\boxed{\approx \pm 0.24 \text{ in.}^2}$$

5) Show that the function $f(x, y) = x^2 - 2x + y$ is differentiable by finding values ε_1 and ε_2 as designated in the definition of differentiability, and verify that both ε_1 and $\varepsilon_2 \to 0$ as $(\Delta x, \Delta y) \to (0,0)$.

$$\boxed{\Delta z = (2x-2)\Delta x + \Delta y + \Delta x(\Delta x) + 0\Delta y,\ \varepsilon_1 = \Delta x,\ \varepsilon_2 = 0}$$

6) Use the function below to show that $f_x(0,0)$ and $f_y(0,0)$ both exist, but that f is not differentiable at $(0,0)$.

$$f(x, y) = \begin{cases} \dfrac{3x^2 y}{x^4 + y^2} & (x, y) \neq (0,0) \\ 0 & (x, y) = (0,0) \end{cases}$$

$f_x(0,0) = 0,\ f_y(0,0) = 0,$ So the partial derivatives exist at $(0,0)$.

Along $y = x$ $\displaystyle\lim_{(x,y)\to(0,0)} f(x, y) = 0$

Along $y = x^2$ $\displaystyle\lim_{(x,y)\to(0,0)} f(x, y) = \frac{3}{2}$

f is not continuous at (0,0), so it is not differentiable at (0,0)

7) Find an equation of the tangent plane to the given surface at the specified point.

a) $z = 4x^2 - y^2 + 2y$; $(-1, 2, 4)$

b) $z = e^{x^2 - y^2}$; $(1, -1, 1)$

a) $\boxed{z = -8x - 2y}$

b) $\boxed{z = 2x + 2y + 1}$

8) Show that that function is differentiable at the given point. Then find the lineararization $L(x, y)$ of the function at that point.

a) $f(x, y) = x\sqrt{y}$; $(1,4)$

b) $f(x, y) = \sin(2x+3y)$; $(-3,2)$

a) f_x and f_y are continuous functions for $y > 0$, f is differentiable at $(1,4)$.

$L(x, y) = 2x + \frac{1}{4}y - 1$

b) f_x and f_y are continuous functions, f is differentiable at $(-3,2)$.

$L(x, y) = 2x + 3y$

9) Find the linear approximation of the function $f(x, y) = \ln(x-3y)$ at $(7,2)$ and use it to approximate $f(6.9, 2.06)$.

$$f(6.9, 2.06) \approx -0.28$$

10) Find the linear approximation of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at $(3,2,6)$ and use it to approximate $f(3.02, 1.97, 5.99)$.

$$f(3.02, 1.97, 5.99) \approx 6.9914$$